Mixed Data Type Exponential Smoothing For Reliability Prediction

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APPLIED MATHEMATICS

UNIVERSITY of WASHINGTON

Overview

- Background
- Current Method
- Proposed Method
- Models
- Simulation
- Analysis
- Summary

Background

- Mathematician for the U.S. Air Force
- Currently Advanced Weapon Systems Analyst
  - ALCM testing and analysis
  - Aircraft nuclear reliability/accuracy for USSTRATCOM



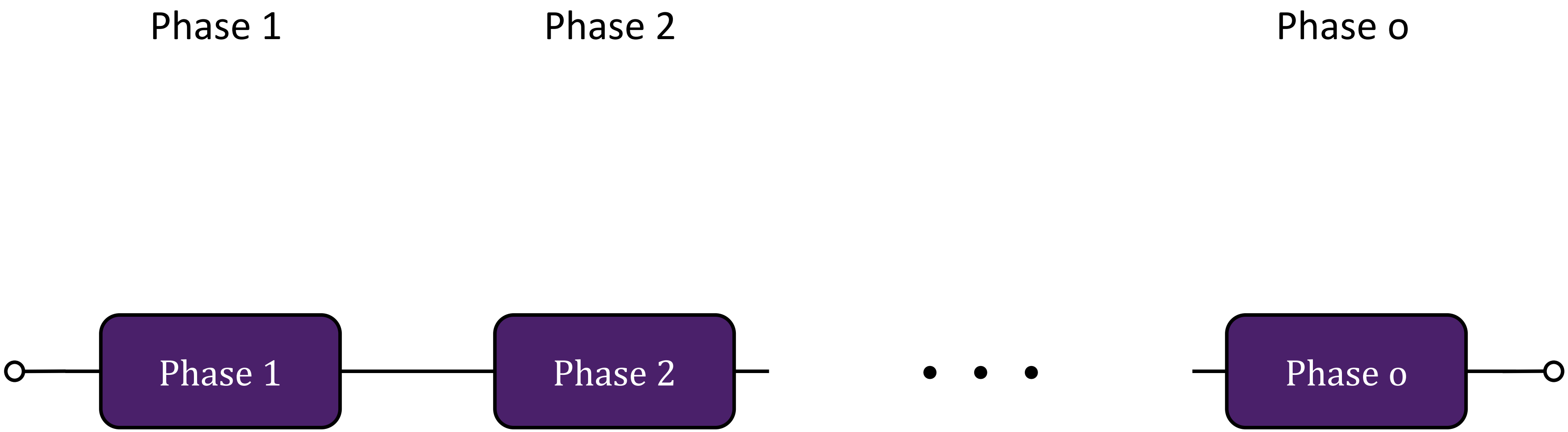


Project Background

- ALCM analysis presents challenges
  - Irregular testing schedule
  - Different types of testing
  - Annual projection required

- Exponential Smoothing selected
  - Accommodates irregular schedule with annual average
  - Allows projection

Overall Model



- System considered as a whole
- Multiple phases of operation

Current Method

- Estimates annual reliability with test success rate
- Form time series from annual reliability estimates
- Simple exponential smoothing to project future reliability

$$P_t = \alpha \overline{R}_{t-1} + (1-\alpha) P_{t-1}$$

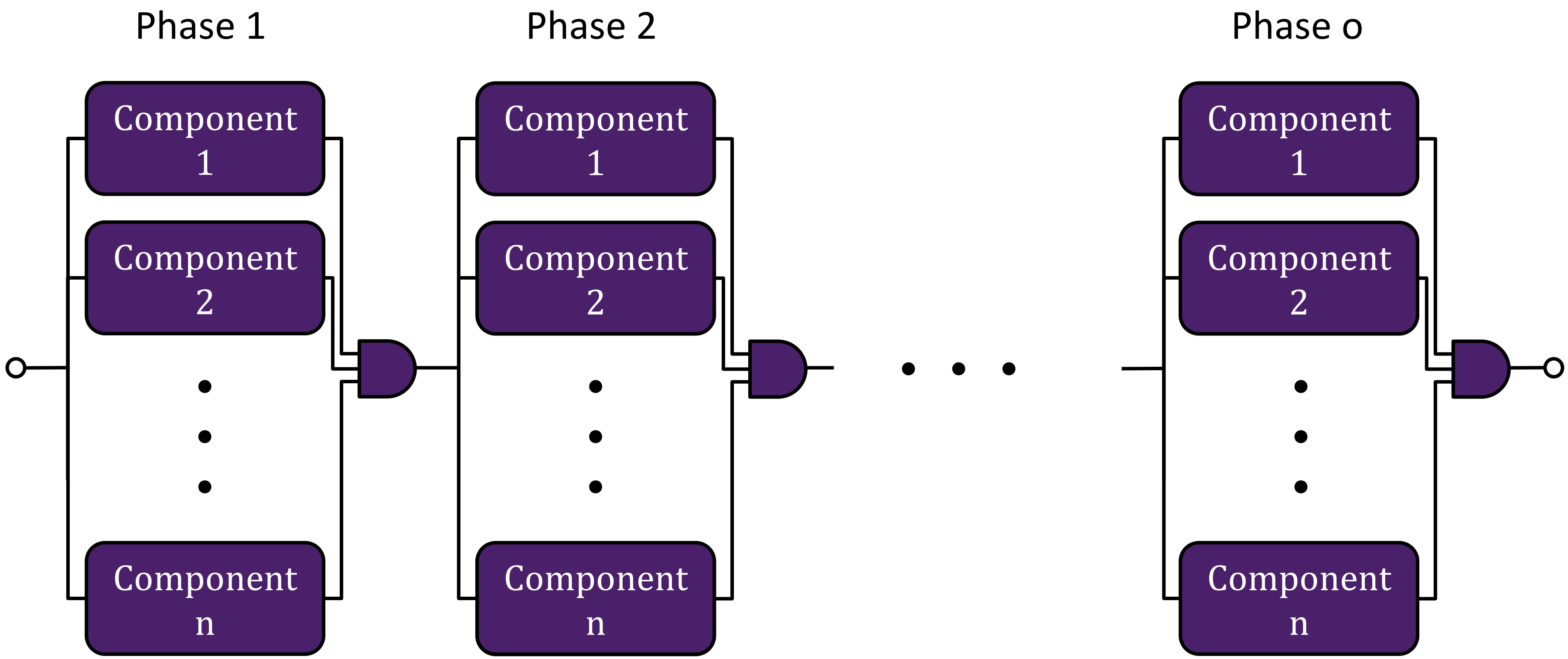
$$t > 1$$

$$\overline{R}_{t-1} = \frac{S_{t-1}}{S_{t-1} + F_{t-1}}$$

Mixed Data Types

- Live fire testing is cost prohibitive
- Other testing stresses different system components more or less than live fire testing
- Need to model reliability at component level to incorporate different types of testing

Overall Model



- Multiple serial components
- Multiple phases of operation

Proposed Method

- Estimates annual reliability with product of component test success rates
- Incorporates mixed types of data

$$P_t = \alpha \overline{R}_{t-1} + (1-\alpha) P_{t-1}$$

$$t > 1$$

$$\overline{R}_{t-1} = \prod_{i=1}^{n} \prod_{j=1}^{o} R_{i,j}$$

Test Weighting

- Relative weights for different types of testing, phases of operation, and components

Test Phase	1					2					…	o				
Component	Test Type					Test Type					…	Test Type				
	FT	1	2	…	m	FT	1	2	…	m	…	FT	1	2	…	m
1	1.0	1.0	1.3	…	0.9	1.0	1.0	0.8	…	0.0	…	1.0	1.0	0.8	…	0.0
2	1.0	0.7	1.0	…	1.0	1.0	0.9	1.5	…	0.5	…	1.0	0.9	1.5	…	0.5
3	0.0	0.0	0.0	…	0.0	1.0	0.2	0.7	…	1.0	…	1.0	0.2	0.7	…	1.0
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
n	1.0	0.9	0.8	…	1.0	1.0	0.9	1.0	…	1.5	…	1.0	0.9	1.0	…	1.5

Model 1

$$R_{i,j} = \frac{S_{i,j,FT}}{S_{i,j,FT} + F_{i,j,FT}}$$

- Flight Testing only
- Control model

Model 2

$$R_{i,j} = \frac{S_{i,j,FT} + S_{i,j,T1} + \cdots + S_{i,j,Tm}}{S_{i,j,FT} + F_{i,j,FT} + S_{i,j,T1} + F_{i,j,T1} + \cdots + S_{i,j,Tm} + F_{i,j,Tm}}$$

- Simple average
- Control model

Model 3

$$R_{i,j} = \frac{S_{i,j,FT} + S_{i,j,T1} \cdot W_{i,j,T1} + \cdots + S_{i,j,Tm} \cdot W_{i,j,Tm}}{S_{i,j,FT} + F_{i,j,FT} + S_{i,j,T1} \cdot W_{i,j,T1} + F_{i,j,T1} + \cdots + S_{i,j,Tm} \cdot W_{i,j,Tm} + F_{i,j,Tm}}$$

- Weighted Successes Model

Model 4

$$R_{i,j} = \frac{S_{i,j,FT} + S_{i,j,T1} + \cdots + S_{i,j,Tm}}{S_{i,j,FT} + F_{i,j,FT} + S_{i,j,T1} + F_{i,j,T1}/W_{i,j,T1} + \cdots + S_{i,j,Tm} + F_{i,j,Tm}/W_{i,j,Tm}}$$

- Weighted Failures Model

Simulation

- Coded in Fortran 90
- Input
  - True component reliabilities for each phase
  - Number of each type of test
- Simulates 100,000 test years
- Compares estimated system reliability to true reliability for each model
- Output
  - Mean error for each model
  - Standard deviation for each model

Simulation Parameters

- Several adjustable parameters for the simulation
  - Number of types of tests **2 - 5**
  - Number of tests of a type **1 - 10**
  - Number of components **30 - 45**
  - Number of phases **1 - 3**
  - Test weights **Varies**
  - True component reliabilities **Varies**
  - True system reliability **~0.5 - ~0.9**

Simulation

```
!--------------------------------------------------------------------------------------------------------------
! Mixed Data Type Exponential Smoothing
! For Reliability Prediction - Model Selection

! Author: Jeremy L. Thompson

! 04 November 2011

! This algorithm compares the average performance of 4 potential models for integrating mixed data types into exponential smoothing
! for reliability prediction.

!--------------------------------------------------------------------------------------------------------------
! Models:

! Model 1: [S_T1] /
!          [S_T1 + F_T1]

! Model 2: [S_T1 + S_T1 + ... + S_Tm] /
!          [S_T1 + F_T + S_T1 + F_T1 + ... + S_Tm + F_Tm]

! Model 3: [S_T1 + S_T2 * W_T2 + ... + S_Tm * W_Tm] /
!          [S_T1 + F_T1 + S_T2 * W_T2 + F_T2 + ... + S_Tm * W_Tm + F_Tm]

! Model 4: [S_T1 + S_T2 + ... + S_Tm] /
!          [S_T1 + F_T1 + S_T2 + F_T2 / W_T2 + ... + S_Tm + F_Tm / W_Tm]

! Notes:
!       Models 1 and 2 are control models
!       Models 3 and 4 are candidate models
!       If a weight factor is 0, then the success or failure of that test is not included
!--------------------------------------------------------------------------------------------------------------

program modelselect

!--------------------------------------------------------------------------------------------------------------
! Part 0: Setup
!--------------------------------------------------------------------------------------------------------------

     !---------------------------------------------------------------------------------------------------------
     ! 0.1 Define variables
     !---------------------------------------------------------------------------------------------------------
     implicit none
     real (kind = 8) :: random, relt, wgtf, wgts
```

Tested Parameter Combinations

	Min	Max
• Extreme combinations	**Min**	**Max**
• Number of types of tests	2	5
• Number of tests of a type	1	10
• Number of components	30	45
• Number of phases	1	3
• Test weights	Varies	
• True component reliabilities	Varies	
• True system reliability	~0.5	~0.9

Tested Parameter Combinations

	Min	Max
• Center combinations		
• Number of types of tests	3	4
• Number of tests of a type	5	5
• Number of components	37	38
• Number of phases	2	2
• Test weights	Varies	
• True component reliabilities	Varies	
• True system reliability	~0.7	~0.7

Analysis Parameters

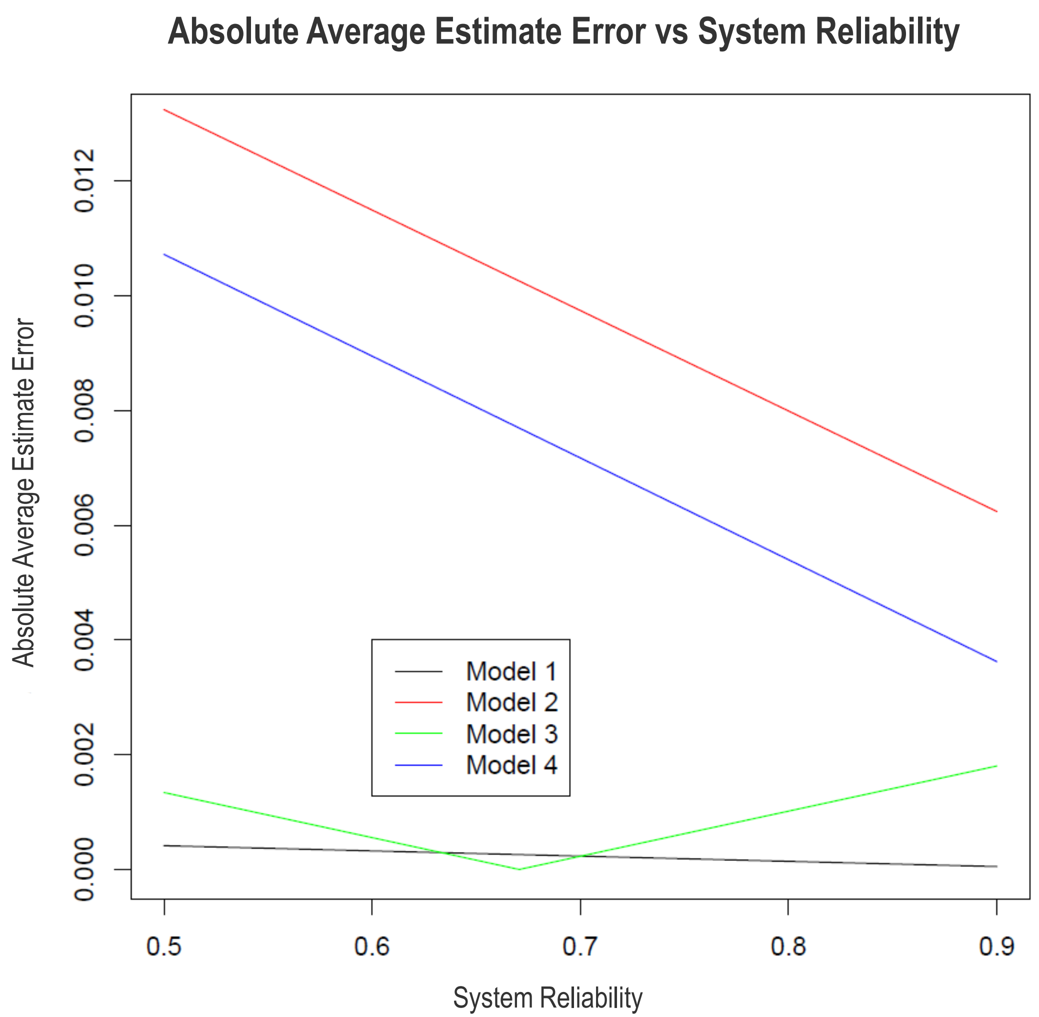
- Parameters were consolidated for analysis
  - Number of types of tests **2 - 5**
  - Number of tests **2 - 50**
  - Ratio of live fire tests to total tests **$0.\overline{09}$ - $0.\overline{90}$**
  - Number of components **30 - 45**
  - Number of phases **1 - 3**
  - True system reliability **0.5 - 0.9**

Analysis

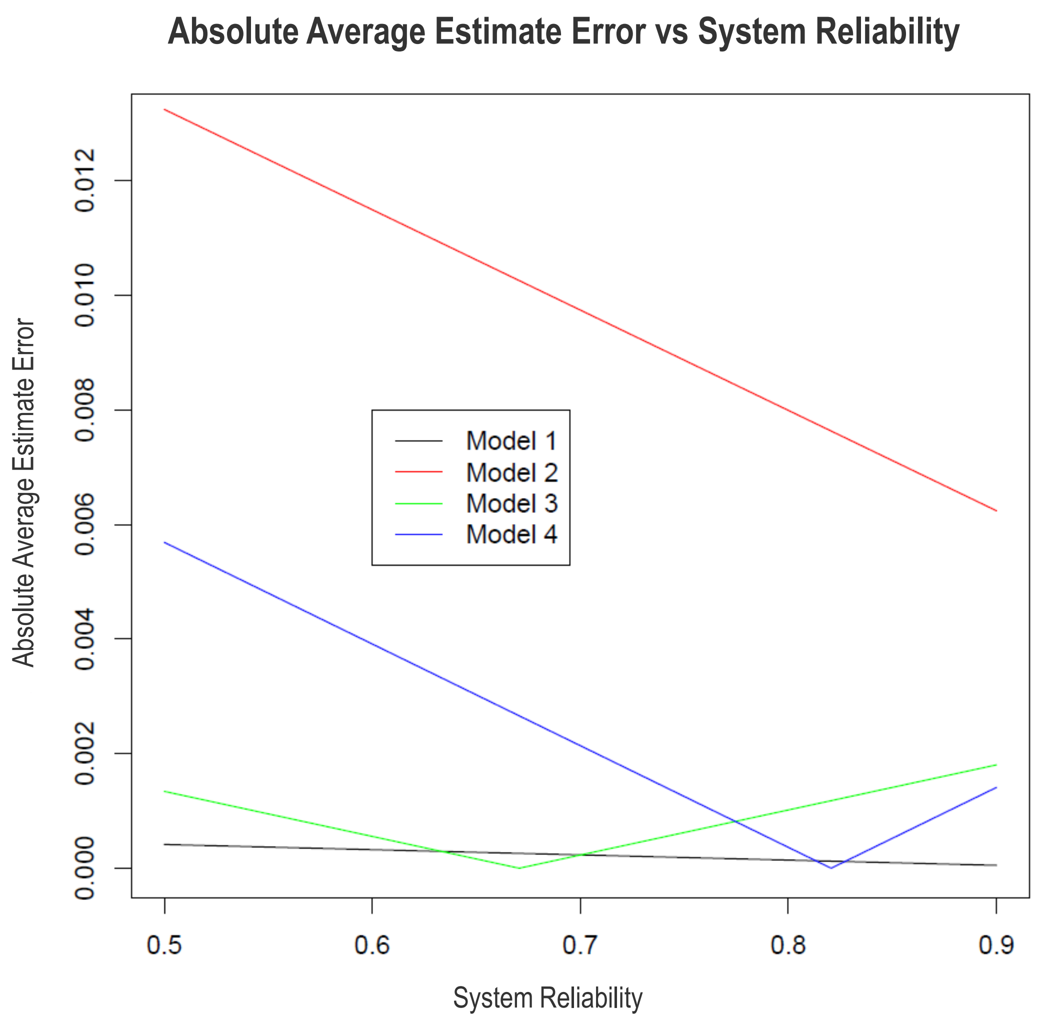
- MANOVA indicates that all parameters except number of phases affect model selection
- Optimal model choice depends upon system under test

```
> summary(model)
                            Df   Pillai    approx F   num Df   den Df   Pr(>F)
data$X..Types.of.Tests      1    0.05368   9.104      4        642      3.707e-07 ***
data$System.Reliability     1    0.64649   293.512    4        642      < 2.2e-16 ***
data$X..Tests               1    0.30713   71.145     4        642      < 2.2e-16 ***
data$Ratio.Live.Total       1    0.38682   101.251    4        642      < 2.2e-16 ***
Residuals              645
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

1 Phase, 2 Types of Test, 20 Tests
0.25 Ratio of Live Fire Testing to Total



1 Phase, 2 Types of Test, 20 Tests 0.75 Ratio of Live Fire Testing to Total



Summary

- Incorporating mixed data types can improve reliability estimates
- Model selection depends upon system under test
  - Simulation should be run for system under test
  - Simulation should be used for sensitivity analysis also

Questions?

Future Research

- Data transformation
- Confidence/prediction bands
  - Take into consideration varying confidence of annual reliability estimates

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